

ASYMPTOTIC MEAN SQUARE ERRORS OF VARIANCE ESTIMATORS FOR U -STATISTICS AND THEIR EDGEWORTH EXPANSIONS

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This paper studies variance estimators for a class of U -statistics. We obtain asymptotic representations of jackknife, Hinkley's (1978) corrected jackknife, unbiased, Sen's (1960) and new variance estimators. And we investigate asymptotic mean square errors of them, theoretically. The Edgeworth expansions of the estimators with remainder term $o(n^{-1})$ are also established. We show that the normalized Hinkley's corrected estimator coincides the normalized unbiased estimator until the order $n^{-1/2}o_p(n^{-1})$.

Key words and phrases: Edgeworth expansions, estimation of variance, jackknife estimator, mean square errors, U -statistics.

1. Sample Mathematics and Text

This short sample document illustrates the typeset appearance of in-line and displayed mathematics in documents. It also illustrates five levels of section headings and three kinds of lists. Finally, the document includes entries for a manual bibliography and an appendix.

1.1. In-line and Displayed Mathematics

The expression $\sum_{i=1}^{\infty} a_i$ is in-line mathematics, while the numbered equation

$$(1.1) \quad \sum_{i=1}^{\infty} a_i$$

is displayed and automatically numbered as equation 1.1.

Let H be a Hilbert space, C be a closed bounded convex subset of H , T a non-expansive self map of C . Suppose that as $n \rightarrow \infty$, $a_{n,k} \rightarrow 0$ for each k , and $\gamma_n = \sum_{k=0}^{\infty} (a_{n,k+1} - a_{n,k})^+ \rightarrow 0$. Then for each x in C , $A_n x = \sum_{k=0}^{\infty} a_{n,k} T^k x$ converges weakly to a fixed point of T [1].

Two sets of \LaTeX parameters govern mathematical displays.* The spacing above and below a display depends on whether the lines above or below are short or long, as shown in the following examples.

A short line above:

$$x^2 + y^2 = z^2$$

and a short line below.

A long line above may depend on your margins

$$\sin^2 \theta + \cos^2 \theta = 1$$

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* \LaTeX automatically selects the spacing depending on the surrounding line lengths.

as will a long line below. This line is long enough to illustrate the spacing for mathematical displays, regardless of the margins.

1.2. Mathematics in section heads $\int_{\alpha}^{\beta} \ln t dt$

Mathematics can appear in section heads. Note that mathematics in section heads may cause difficulties in typesetting styles with running headers or table of contents entries.

1.3. Theorems, Lemmata, and Other Theorem-like Environments

A number of theorem-like environments is available. The following lemma is a well-known fact on differentiation of asymptotic expansions of analytic functions.

Lemma 1. *Let $f(z)$ be an analytic function in \mathbb{C}_+ . If $f(z)$ admits the representation*

$$f(z) = a_0 + \frac{a_1}{z} + o\left(\frac{1}{z}\right),$$

for $z \rightarrow \infty$ inside a cone $\Gamma_{\varepsilon} = \{z \in \mathbb{C}_+ : 0 < \varepsilon \leq \arg z \leq \pi - \varepsilon\}$ then

$$(1.2) \quad a_1 = -\lim_{z \rightarrow \infty, z \in \Gamma_{\varepsilon}} z^2 f'(z).$$

Proof. Change z for $1/z$. Then $\Gamma_{\varepsilon} \rightarrow \bar{\Gamma}_{\varepsilon} = \{z \in \mathbb{C}_- : \bar{z} \in \Gamma_{\varepsilon}\}$ and

$$(1.3) \quad f(1/z) = a_0 + a_1 z + o(z).$$

Fix $z \in \bar{\Gamma}_{\varepsilon}$, and let $C_r(z) = \{\lambda \in \mathbb{C}_- : |\lambda - z| = r\}$ be a circle with radius $r = |z| \sin \varepsilon/2$. It follows from (1.3) that

$$(1.4) \quad \frac{1}{2\pi i} \int_{C_r(z)} \frac{f(\lambda) d\lambda}{(\lambda - z)^2} = \sum_{m=0}^1 a_m \frac{1}{2\pi i} \int_{C_r(z)} \frac{(\lambda - z_0)^m d\lambda}{(\lambda - z)^2} + R(z),$$

where for the remainder $R(z)$ we have

$$\begin{aligned} |R(z)| &\leq r^{-1} \max_{\lambda \in C_r(z)} o(|z|) = r^{-1} \max_{\lambda \in C_r(z)} |\lambda| \cdot O(|z| + r) \\ &= \frac{|z| + r}{r} \cdot O(|z| + r) = \frac{1 + \sin \varepsilon}{\sin \varepsilon} \cdot O(|z|). \end{aligned}$$

Therefore $R(z) \rightarrow 0$ as $z \rightarrow \infty, z \in \bar{\Gamma}_{\varepsilon/2}$, and hence by the Cauchy theorem (1.4) implies

$$\frac{d}{dz} f(1/z) = a_1 + R(z) \rightarrow a_1, \text{ as } z \rightarrow \infty, z \in \bar{\Gamma}_{\varepsilon/2},$$

that implies (1.2) by substituting $1/z$ back for z . ■

2. Section Headings

Use the Section tag for major sections, such as the one just above. Four additional heading levels are available, as described below.

2.1. Subsection Heading

This text appears under a subsection heading.

2.1.1. Subsubsection Heading

This text appears under a subsubsection heading.

Subsubsubsection Heading This text appears under a subsubsubsection heading.

Subsubsubsubsection Heading This text appears under a subsubsubsubsection heading.

3. Lists

Bullet, numbered and description list environments are available. Lists, which can extend four levels deep, look like this:

- (i) Numbered list item 1.
- (ii) Numbered list item 2.
 - (a) A numbered list item under a list item.

The typeset appearance for this level is often different from the screen appearance. The typeset appearance often uses parentheses around the level indicator.
 - (b) Another numbered list item under a list item.
 - i. Third level numbered list item under a list item.
 - A. Fourth and final level of numbered list items allowed.
- Bullet item 1.
- Bullet item 2.
 - Second level bullet item.
 - * Third level bullet item.
 - Fourth and final level bullet item.

Description List Each description list item has a lead-in followed by the item. Double-click the lead-in box to enter or customize the text of the lead-in.

Bunyip Mythical beast of Australian Aboriginal legends.

4. Tags

You can apply the logical markup tag *Emphasized*.

You can apply the visual markup tags **Bold**, *Italics*, Roman, Sans Serif, *Slanted*, SMALL CAPS, and Typewriter.

You can apply the size tags `tiny`, `scriptsize`, `footnotesize`, `small`, `normalsize`, `large`, `Large`, `LARGE`, `huge` and `Huge`.

You can apply `BLACKBOARD BOLD`, *CALLIGRAPHIC*, and `fraktur`, the special, mathematics-only, tags. Note that blackboard bold and calligraphic are correct only when applied to uppercase letters A through Z.

Following is a group of paragraphs marked as Short Quote. This environment is appropriate for a short quotation or a sequence of short quotations.

The only thing we have to fear is fear itself. *Franklin D. Roosevelt*, Mar. 4, 1933

Ask not what your country can do for you; ask what you can do for your country. *John F. Kennedy*, Jan. 20. 1961

There is nothing wrong with America that cannot be cured by what is right with America. *William J. "Bill" Clinton*, Jan. 21, 1993

5. About the Bibliography

Following the text of this article is a short manual bibliography. This sample bibliography has no relationship to the previous text, but it shows sample citations such as [4], [5] and [6]. You can also have multiple citations appear together. Here is an example: [2, 3, 4].

REFERENCES

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