Title of the Book

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Preface

Preface Head

This is the preface and it is created using a TeX field in a paragraph by itself. When the document is loaded, this appears in the edit window like a normal chapter, but it is actually an unnumbered chapter. The TeX field at the beginning of this paragraph sets the correct page heading for the Preface portion of the document. The preface does not appear in the table of contents.

Acknowledgments

Scientific Notebook is the latest product conceived by Roger Hunter, President of *TCI Software Research*, and his staff, in a series of electronic tools developed for the teaching, learning, and applying of mathematics. *Scientific Notebook* is a natural electronic blackboard and a syntax-free interface to the *Maple* symbolic computation system. It is an extension of *Scientific WorkPlace,* a TeX-based document production system. *Scientific Notebook* is elegant, easy, and powerful. It is fun to work with and makes mathematics, both elementary and advanced, accessible to a wider audience than ever before. I feel privileged to be able to use *Scientific Notebook* to make linear algebra accessible, enjoyable, and rewarding. Thank you, Roger, for your genius, dedication, and persistence.

XYZ University   *Author*.

Contents

1 Chapter Head

About the Shell

This style is based on the swugbk style. It involves the following modifications:

- The crop marks are turned off.
- The index prints in two columns instead of three.
- Definitions constitute an additional theorem-like environment.
- All definitions, theorems, etc. are numbered within chapters.
- A solution label is defined in the preamble.
- A Maple output environment is defined in the preamble.
- Additional white space is introduced between the rows of matrices for easier readability of fractional entries.
- Fonts and screen tags, in particular screen colors, are used to enhance readability.

About the Preamble

The preamble of this shell includes the following items:

"input–tcilatex"
"def"solution–"noindent–"bfseries Solution. ""–"
"newenvironment–MapleBox"–"STARTGRAYBOX"–"ENDGRAYBOX"
"renewcommand–"arraystretch"–1.25"

The first item loads tcilatex. The second one defines the solution label. The third one defines the environment used to display the result of an evaluation. It renders the result in yellow on screen and light grey in print. The last item inserts additional space between the rows of a matrix. It makes fractional entries more readable.

2 Chapter Head

Mathematics Examples

Relevant material

Example 2.1 ■*Example*   *A Suitable Problem*

Each example has a label, followed by a descriptive title. Examples are numbered within chapters.

Solution. Solutions are introduced by the solution label.

All definitions are preceded by the definition label. Definitions are numbered within chapters.

A linear equation in the variables $x_1, \ldots, x_n$ over the field $\mathbb{R}$ is an equation of the form
$$a_1 x_1 + \cdots + a_n x_n = b,$$
where the coefficients $a_1, \ldots, a_n$ and the constant b are in $\mathbb{R}$.

More Examples

Example 2.2 ■ *Example*   *A Linear System with a Unique Solution*

Use *Scientific Notebook* to find a solution for the system
$$\begin{cases} x + y - z - 4 = 0 \\ 2x - y - 3z + 6 = 0 \\ -x + y + 6z = 0 \end{cases}.$$

Solution. The following command produces a solution for the system.

This is a Maple Box showing the result of a Scientific Notebook computation.

$$\begin{cases} x + y - z - 4 = 0 \\ 2x - y - 3z + 6 = 0 \\ -x + y + 6z = 0 \end{cases}, \text{ Solution is : } \left\{ x = -\frac{30}{13}, z = -\frac{16}{13}, y = \frac{66}{13} \right\}$$

Exercises

Note This is a note. Notes are unnumbered.

Lemma 2.1 *Here is a lemma. Lemmas are numbered within chapters.*

Corollary 2.2 *The matrix T has can be represented as a direct sum*

$$C_1 \oplus \cdots \oplus C_r,$$

of companion matrices for a unique r, with C_i being the companion matrix of the T-annihilator of $f(t)^{n_i}$. Corollaries are numbered within chapters.

Theorem 2.3 (Rational Form Theorem) *Suppose that $T : V \rightarrow V$ is a linear transformation whose minimum polynomial over the field $\Bbbk$ of the space V is the product of the distinct irreducible monic factors $p_1(t)^{n_1}, \ldots, p_1(t)^{n_{r_1}}$. Then T can be represented as the direct sum*

$$(C_{11} \oplus \cdots \oplus C_{1n_1}) \oplus \cdots \oplus (C_{r1} \oplus \cdots \oplus C_{rn_r})$$

of companion matrices. Each matrix $C_{i1} \oplus \cdots \oplus C_{in_i}$ is a direct sum of the form described in Lemma 2.1.

Proof. See Appendix A. ∎

Theorems are numbered within chapters.

A   Additional Material

Mathematical Induction

Mathematical induction is an accepted procedure for proving that certain statements concerning the natural numbers are true no matter which natural number is involved. For example, we might have a formula $\varphi(n)$ for computing the sum $1 + 2 + \cdots + n$, and we want to prove that the formula works for all n. If $n = 15$, then the formula $\varphi(15)$ computes the sum $1 + 2 + \cdots + 15$, if $n = 1001$, then $\varphi(1001)$ compute the sum $1 + 2 + \cdots + 1001$, and so on. We would *prove* the fact that we can use the formula $\varphi(n)$ in this way by using mathematical induction.

The reason behind mathematical induction is the fact that, as discussed above, the set $\mathbb{N}$ of natural numbers is characterized by Peano's axioms as the set each of whose elements is either 0 or of the form $n + 1 = (1 + 1 + \cdots + 1) + 1$. If we can prove that $\varphi(0)$ holds, and if we can deduce the validity of $\varphi(n + 1)$ from the validity of $\varphi(n)$, then the principle of mathematical induction asserts that the statement $\varphi(m)$ holds for all $m \in \mathbb{N}$.

It is clear from the given illustration, that a proof by mathematical induction consists of two parts:

1. The proof that $\varphi(0)$ holds. This part is called the *induction basis*. Depending on the specific claim expressed by the formula $\varphi(n)$, the statement may actually only make sense for $p > 0$. In that case, the induction basis is the proof that $\varphi(p)$ holds. We may also have to prove the validity of the first case or two by different means. This depends on the nature of the formula $\varphi(n)$. If the induction basis involves more than one initial proof, the induction involved is known as *complete induction*.

2. The proof that if $\varphi(q)$ holds for an arbitrary $q \in N$, then $\varphi(q + 1)$ holds. This part is called the *induction step*.

Note The use of the word *basis* in mathematical induction has nothing to do with the use of the word *basis* in linear algebra. It will of course always be clear from the context which meaning of the word is intended.

The following example illustrates the two parts involved in a proof by mathematical induction. It is well-known (and proved by mathematical induction) that

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2} = \varphi(n)$$

for all $n \in \mathbb{N}$. We would prove $\varphi(2)$ by observing that, indeed,

$$1 + 2 = 3 = \frac{2(2+1)}{2}.$$

We would then show how we can deduce the validity of $\varphi(q+1)$ from the assumed validity of $\varphi(q)$:

Suppose that

$$1 + 2 + \cdots + q = \frac{q(q+1)}{2}.$$

Then

$$
\begin{aligned}
1 + 2 + \cdots + q + (q+1) &= \frac{q(q+1)}{2} + (q+1) \\
&= \frac{q(q+1)}{2} + \frac{2(q+1)}{2} \\
&= \frac{(q+2)(q+1)}{2} \\
&= \frac{(q+1)(q+2)}{2}.
\end{aligned}
$$

The principle of mathematical induction therefore allows us to conclude that $\varphi(n)$ holds for all n. (In this case all $n \geq 2$, since the expression makes no sense for $n = 0$ or $n = 1$.)

In finite-dimensional linear algebra, statements of the form $\varphi(n)$ are usually statements asserting that a certain fact φ hold for spaces of all dimensions $n \in \mathbb{N}$. Since there is a space of dimension 0, we would have to prove that $\varphi(0)$ holds. We may also have to prove, in addition, that $\varphi(1)$ holds, if the validity of $\varphi(1)$ does not follow from that of $\varphi(0)$, given the triviality of the space $V = \{\mathbf{0}\}$. Other types of proofs in linear algebra using mathematical induction are proofs about linear combinations of arbitrary length, or polynomials of arbitrary degrees.

Sigma Notation

Another important application of mathematics induction is the definition of the "sigma notation." Arithmetical statements in linear algebra are often about sums made up of a fixed but "arbitrary" number of terms. There are two basic ways of dealing with this situation. We can either use an *ellipsis* (three dots) to represent missing elements as in $\mathbf{v}_1 + \cdots + \mathbf{v}_n$, or we can use variable subscripts and the capital sigma and write

$\sum_{i=1}^{n} \mathbf{v}_i$. Both expressions have the same meaning. The sigma notation is defined by mathematical induction.

Proposition 2.4   *If $n = 1$, then $\sum_{i=1}^{n} \mathbf{v}_i = \mathbf{v}_1$, and if $n = k + 1$, then $\sum_{i=1}^{n} \mathbf{v}_i = \left(\sum_{i=1}^{k} \mathbf{v}_i\right) + \mathbf{v}_n$.*

In this text, the expression $\sum_{i=1}^{n} \mathbf{v}_i$ is usually written less formally as "$v_1 + \cdots + v_n$" using the *ellipsis* "$\cdots$". Although the use of ellipses is avoidable in the case of $\sum_{i=1}^{n} \mathbf{v}_i$, there are many situations in linear algebra where ellipses are an essential part of the notation. For example, we frequently need to refer to *finite sequences* of vectors of unspecified length n, such as $\mathbf{v}_1, \ldots, \mathbf{v}_n$, and to *finite arrays* of elements of unspecified dimensions m and n, such as

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mm} \end{bmatrix}.$$

In these situations, horizontal, vertical, and diagonal ellipses are used to indicate possibly missing terms.

B References

This appendix contains the references.

C Index

This appendix contains the index.